Research Paper | Physics | E-ISSN No : 2454-9916 | Volume : 11 | Issue : 2 | February 2025

EQUILIBRIUM CHARGE DISTRIBUTION IN THE SPACE BETWEEN A CONDUCTING WIRE AND A CO-AXIAL METAL CYLINDER IN CLASSICAL APPROXIMATION: A PEDAGOGICAL COMMUNICATION

Dr. Palash Das

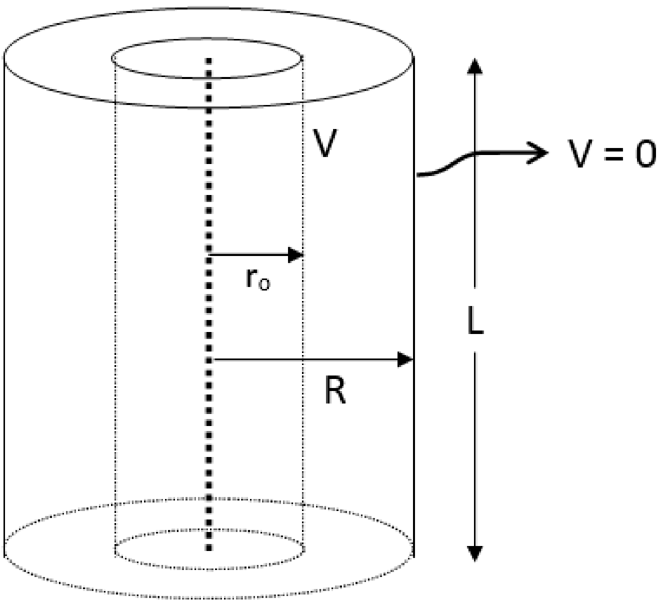
Department of Physics, Santipur College, Santipur, West Bengal, India

ABSTRACT

We consider a classical electron gas filling the space between a conducting wire and a metallic cylinder. In this pedagogical communication, we shall investigate the functional dependence of the electric charge density on the radial distance *r* from the wire in thermal equilibrium and also analyse its experimental importance giving a statistical interpretation.

INTRODUCTION

Let us consider a wire of radius r_0, placed along the axis of a metal cylinder of radius R and length L. The wire is maintained at a positive potential V with respect to the cylinder and the whole system is maintained at some high absolute temperature T. As a result, electrons emitted from the metals form a dilute gas filling the cylindrical container and in equilibrium with it. The density is low enough to neglect the electrostatic interaction between the electrons. We also assume the cylinder to be long enough $(L \gg R)$ to neglect the end effects.



We now study the dependence of the electrostatics potential and the electric charge density on the radial distance r $(r_0 < r < R)$ in thermal equilibrium. For this, we recall the Gauss' theorem in electrostatics, according to which

$$\oint_S \vec{E} \cdot d\vec{S} = 4\pi q_{enclosed} \tag{1}$$

where $\vec{E}(\vec{r})$ is the electric field at a radial distance $\vec{r}$ and $q_{enclosed}$ is the charge enclosed by the Gaussian surface, shown above. Using Gauss' divergence theorem, Eq.(1) can be written as

$$\vec{\nabla}\cdot\vec{E}(\vec{r}) = 4\pi\rho(\vec{r}) \tag{2}$$

where $\rho(\vec{r})$ is the volume charge density.

The charge density is very low as assumed earlier and so to a first approximation, we may take $\rho(\vec{r}) \cong 0$ and then Eq.(2) becomes

$$\vec{\nabla}\cdot\vec{E}(\vec{r}) = 0 \Rightarrow \nabla^2 V(r) = 0 \Rightarrow \frac{1}{r}\frac{d}{dr}\left(r\frac{dV}{dr}\right) = 0 \Rightarrow r\frac{dV}{dr} = const.(say, C) \Rightarrow V(r) = C\ln r + D$$

where D is another constant of integration.

The boundary conditions give

$$V = C\ln r_0 + D$$
$$0 = C\ln R + D$$

Solving the above equations, we get

$$C = \frac{V}{\ln\frac{r_0}{R}} \text{ and } D = -\frac{V\ln R}{\ln\frac{r_0}{R}} \tag{3}$$

On substituting,

$$V(r) = C\ln\frac{r}{R} \Rightarrow V(r) = \frac{V ln\frac{r}{R}}{ln\frac{r_0}{R}} \tag{4}$$

This is the electrostatic potential at a radial distance r from the wire.

The charge density at a radial distance r, according to classical statistics, is given by

$$\langle\sigma(r)\rangle \propto \frac{Ne}{\Omega}\frac{e^{-\beta V(r)}}{\int e^{-\beta V(r)} d^3r} \tag{5}$$

where $\beta = \frac{1}{k_B T}$, N = the total number of electrons forming the gas, Ω = the volume of the entire space between the wire and the cylinder and e = the charge of an electron.

But $e^{-\beta V(r)} = e^{-\beta C\ln\frac{r}{R}} = \left(\frac{r}{R}\right)^{-\beta C}$ (6)

$$\therefore \langle\sigma(r)\rangle \propto \frac{\left(\frac{r}{R}\right)^{-\beta C}}{\int\left(\frac{r}{R}\right)^{-\beta C} d^3r} = \frac{r^{-\beta C}}{R^{-\beta C}\cdot R^{\beta C}} \times \frac{1}{\underbrace{\int_{r_0}^{R} r^{-\beta C}\, 2\pi r L dr}_{a\ constant}} \propto r^{-\beta C}$$

Using Eq.(3), we finally get the distribution of charge density as

$$\langle \sigma(r) \rangle \propto r^{-\beta \frac{V}{\ln \frac{r_0}{R}}} \tag{7}$$

This is the result we want to show.

In closing, we note that the above distribution leads to a practical method of obtaining the radius (r_0) of a wire. By determining the charge densities $\langle \sigma(r_1) \rangle$ and $\langle \sigma(r_2) \rangle$ at two radial distances r_1 and r_2 respectively, we find from Eq.(7)

$$ln \frac{r_0}{R} = \frac{V}{k_B T} \times \frac{ln r_2 - ln r_1}{ln\langle \sigma(r_1) \rangle - ln\langle \sigma(r_2) \rangle} \quad (r_2 > r_1)$$

or

$$r_0 = R exp\left[\frac{V}{k_B T} \times \frac{ln r_2 - ln r_1}{ln\langle \sigma(r_1) \rangle - ln\langle \sigma(r_2) \rangle}\right] \tag{8}$$

This equation expresses r_0 in terms of charge density. So knowing experimentally $\langle \sigma(r_1) \rangle$ and $\langle \sigma(r_2) \rangle$, one can evaluate r_0.

REFERENCES

1. Kerson Huang, 'Statistical Mechanics', Wiley Eastern Limited.
2. Federick Reif, 'Fundamentals of Statistical and Thermal Physics', McGraw-Hill Book Company.
3. R.K. Pathria, 'Statistical Mechanics', Butterworth-Heinemann.
4. Mark W. Zemansky and Richard H. Dittman, 'Heat and Thermodynamics', McGraw-Hill Book Co.